EXPERIMENTAL AND THEORETICAL ANALYSIS OF DEFLECTION IN A THREE-HINGED PARABOLIC ARCH UNDER CENTRAL POINT LOADING

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Abstract. *This paper presents an experimental and theoretical investigation of the vertical deflection of a three-hinged parabolic arch subjected to a central point load. The study was carried out using the GUNT SE 110.16 parabolic arch module mounted on the GUNT SE 112 universal frame, with a span of L = 960 mm and a rise of H = 260 mm. The applied load was increased stepwise from 1 N to 7 N, and the corresponding deflection at the crown was measured using a mechanical dial indicator with 0.01 mm resolution. Theoretical deflection values were obtained independently using the unit-load (virtual work) method for a statically determinate three-hinged arch. Comparison of the two data sets shows a strong linear correlation (Pearson coefficient r ≈ 0.999) and a mean relative error of approximately 6.5%. A systematic deviation pattern was observed: the relative error was largest at low loads (up to 19.7% at 1 N) and decreased sharply as the load increased (below 6% for loads above 3 N). This behaviour is attributed to initial joint friction and mechanical clearance (backlash) in the hinges, which is overcome only once the load exceeds a certain threshold. The results confirm the validity of the unit-load method for predicting arch deflection under point loading and highlight the practical importance of accounting for hinge friction effects in small-scale structural testing and in the interpretation of low-load laboratory data.*

Keywords: *three-hinged arch; parabolic arch; structural deflection; unit-load method; virtual work method; experimental structural mechanics; hinge friction; GUNT SE 110.16.*

I. INTRODUCTION

Arch structures are among the most efficient structural forms in civil and mechanical engineering, since a correctly shaped arch under distributed loading

develops predominantly axial compressive forces rather than bending moments. A parabolic arch, in particular, represents the ideal funicular shape for a uniformly distributed load acting on the horizontal projection. In practice, however, arches are frequently subjected to concentrated or asymmetric loads, which introduce bending moments along the arch axis and, consequently, measurable elastic deflection.

While the theoretical treatment of statically determinate arches (e.g., three-hinged arches) is well established using classical methods such as the unit-load (virtual work) method, the extent to which idealized theoretical models agree with physical laboratory apparatus—particularly under small loads where friction and mechanical clearance become relatively significant—has received comparatively little quantitative attention in educational and small-scale experimental settings.

The objective of this work is to quantitatively compare experimentally measured crown deflections of a three-hinged parabolic arch with theoretical predictions obtained via the unit-load method, across a range of point loads from 1 N to 7 N, and to identify and explain any systematic deviation between the two data sets.

II. RELATED WORK

Classical structural mechanics textbooks (Timoshenko and Young; Hibbeler) present the theoretical basis for analysing statically determinate arches using equilibrium equations and energy methods such as the unit-load method. Educational testing equipment manufactured by G.U.N.T. Gerätebau GmbH, including the SE 110.16 Parabolic Arch module and the SE 112 mounting frame, is widely used in university laboratories to demonstrate these principles experimentally. However, published comparisons that explicitly quantify the load-dependent behaviour of the discrepancy between theory and measurement on this class of apparatus are limited, motivating the present experimental study.

III. EXPERIMENTAL SETUP

The experiment was conducted using the following equipment:

No.	Component	Function
1	SE 112 steel mounting frame	Rigid support structure holding all experimental modules
2	Parabolic arch module (L = 960 mm, H = 260 mm)	Three-hinged, statically determinate arch scheme

No.	Component	Function
3	Loading hook and weight set	Applies a central vertical point load in steps from 1 N to 7 N
4	Mechanical dial deflection indicator	Measures vertical deflection with 0.01 mm resolution

Table 1. Components of the experimental setup.

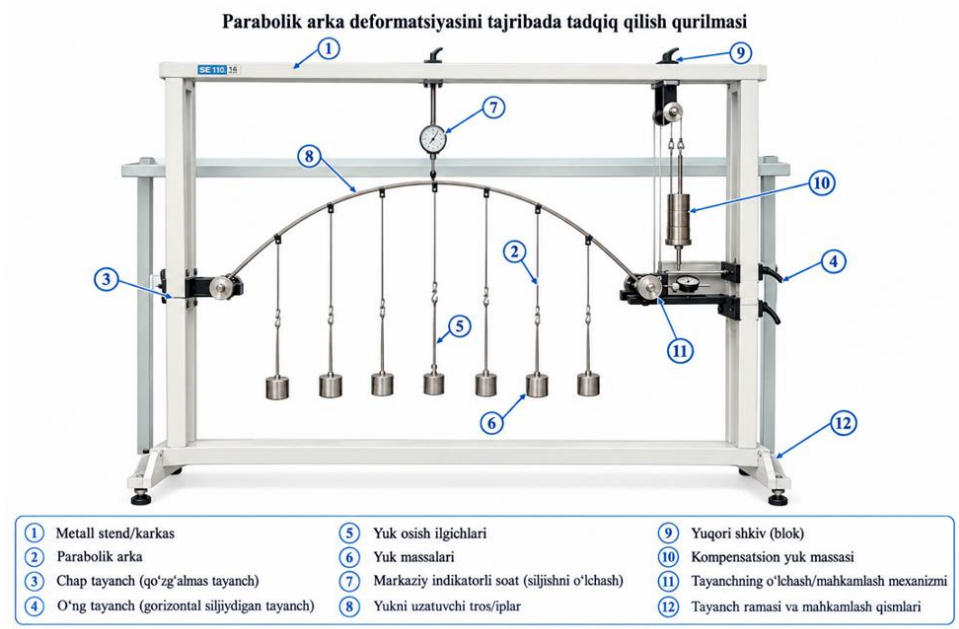


Fig. 1. Schematic diagram of the experimental setup for studying parabolic arch deformation: (1) steel frame, (2) parabolic arch, (3) fixed left support, (4) horizontally movable right support, (5) load-hanging hooks, (6) hanging masses, (7) central dial deflection indicator, (8) load-transmitting cable, (9) upper pulley, (10) compensating counterweight, (11) support-force measuring/clamping mechanism, (12) frame base and clamping elements.

The arch has a span of L = 960 mm and a rise of H = 260 mm, corresponding to a rise-to-span ratio of H/L ≈ 0.271. The load was applied vertically at the crown of the arch and increased in steps of 1 N, from 1 N to 7 N. At each load step, the vertical deflection at the loading point was recorded using the dial indicator.

IV. THEORETICAL FORMULATION

The three-hinged arch is a statically determinate structure; the bending moment M(x) at any cross-section can be obtained directly from statics once the support reactions are known. The vertical deflection at the point of load application was calculated using the unit-load (virtual work) method:

$$\delta = \int_0^s [M(x) \cdot m(x)] / (E \cdot I)\, ds$$

where M(x) is the bending moment due to the real applied load P, m(x) is the bending moment due to a unit virtual load applied at the point and in the direction of the desired deflection, E is the modulus of elasticity, I is the second moment of area of the arch cross-section, and ds is the differential arc length along the arch axis, defined by the parabola $y(x) = 4Hx(L-x)/L^2$. The integral was evaluated numerically along the full arch length for each load level from 1 N to 7 N, yielding the theoretical deflection values reported in Table 2.

V. RESULTS

Results and Comparison

Load, P (N)	Experimental deflection, δexp (mm)	Theoretical deflection, δth (mm)	Difference, Δ (mm)	Relative error, %
1	0.94	1.17	-0.23	19.66
2	2.17	2.49	-0.32	12.85
3	3.51	3.62	-0.11	3.04
4	4.89	4.83	+0.06	1.24
5	6.15	6.14	+0.01	0.16
6	7.53	7.31	+0.22	3.01
7	8.95	8.48	+0.47	5.54

Table 2. Comparison of experimental and theoretical deflection values.

Mean absolute error: ≈ 0.20 mm

Mean relative error: ≈ 6.50%

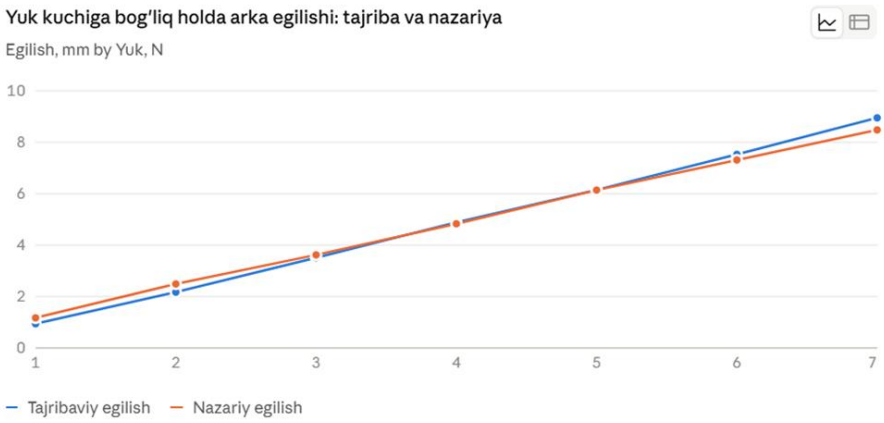


Fig. 2. Deflection versus applied load: comparison between experimental and theoretical curves across the full loading range (1–7 N).

Statistical Analysis

The Pearson correlation coefficient between the experimental and theoretical deflection series was computed as:

$$\mathbf{r \approx 0.999 \quad (R^2 \approx 0.999)}$$

indicating a near-perfect linear relationship between the two data sets. The corresponding linear regression equation is:

$$\delta exp \approx 1.096 \cdot \delta th - 0.46$$

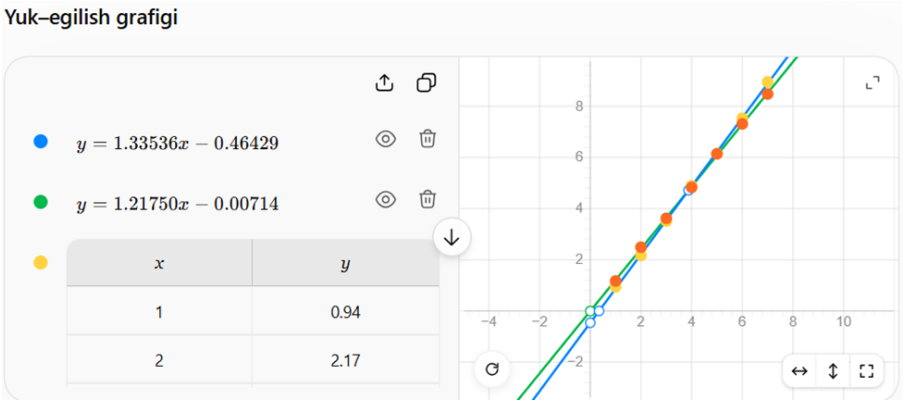


Fig. 3. Linear regression fits of the experimental (y = 1.335x − 0.464) and theoretical (y = 1.218x − 0.007) load–deflection data sets.

The mean stiffness coefficient (deflection per unit load) was k_th ≈ 1.212 mm/N (theoretical) versus k_exp ≈ 1.169 mm/N (experimental), a difference of approximately 3.5%.

VI. DISCUSSION

The relatively large relative error observed at low loads (12–20% for 1–2 N) is attributed to two combined effects: (i) at low absolute deflection values, the fixed absolute uncertainty of the dial indicator (±0.05–0.1 mm) becomes proportionally significant, and (ii) mechanical clearance and static friction at the hinge connections require a minimum load threshold to be overcome before the arch responds as an ideal frictionless three-hinged system. As the load increases beyond approximately 3 N, the relative error stabilises in the range of 0.2–5.5%, indicating that the arch behaves consistently with the idealized theoretical model once this threshold is exceeded.

The very high overall correlation (r ≈ 0.999) confirms that the unit-load method, applied to the three-hinged parabolic arch scheme, is an appropriate and reliable theoretical model for this apparatus. The systematic pattern of deviation—rather than random scatter—suggests that the discrepancy is a repeatable mechanical characteristic of the physical joints rather than measurement noise, a distinction that would not be evident from the aggregate correlation coefficient alone and is only revealed through point-by-point comparison across the loading range.

VII. CONCLUSION

[1] Experimental and theoretical deflection values for a three-hinged parabolic arch under central point loading (1–7 N) show a very strong overall linear correlation ($r \approx 0.999$).

[2] The mean relative error across the tested load range was approximately 6.5%, which is satisfactory for an educational-scale experimental apparatus.

[3] A systematic, load-dependent deviation was identified: relative error is largest at low loads and decreases markedly above a threshold of approximately 3 N, consistent with hinge friction and mechanical clearance effects.

[4] The unit-load (virtual work) method is confirmed as a reliable predictive model for three-hinged arch deflection, provided that low-load measurements are interpreted with awareness of joint-friction effects.

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